

ICMT Common Mathematical Notions

1. Answers should be exact and simplified. For more information on valid answer formats, see the Acceptable Answer Formats document.
2. If a diagram is given with a problem, it is not necessarily drawn to scale.
3. In a triangle ABC , the vertices are called A , B , and C . The sides are called a , b , and c , with side a opposite vertex A , side b opposite vertex B , and side c opposite vertex C . If a polygon is called $ABCDEF$, its vertices will occur in that order around the polygon. This convention holds for all namings of polygons.
4. Unless otherwise noted, polygons (including triangles) are simple and non-degenerate.
5. If complex numbers are used in a problem, i denotes $\sqrt{-1}$.
6. \mathbb{Z} denotes the set of integers, \mathbb{Q} the set of rational numbers, \mathbb{R} the set of real numbers, and \mathbb{C} the set of complex numbers.
7. Logs are base e unless otherwise indicated. When logs are used in a different base, a subscript will be used, as in $\log_{10} 2$. Base e logs may also be written with \ln , as in $\ln 2$.
8. The exponential function e^x may also be denoted $\exp(x)$.
9. The word *prime* refers to positive integers only. Note that 1 is not a prime.
10. Divisors and factors of a positive integer refer to positive numbers only. Proper divisors of a positive integer refer to divisors that are less than that integer.
11. The notation $|S|$ denotes the number of elements in a finite set S .
12. If A and B are sets, the notation $A \subseteq B$ means that every element of A is also an element of B . In this case, A is called a subset of B . If additionally $A \neq B$, then A is called a proper subset of B , and this may be denoted by $A \subset B$.
13. If a problem refers to the digits of a number, those digits are underlined to distinguish the digits of a number from the product of the digits. For example, $\underline{3}\underline{1}\underline{A}\underline{B}$ refers to a four digit number and not the product $3 \cdot 1 \cdot A \cdot B$.
14. Combinations will be denoted by $\binom{n}{k}$; this is the number of ways to choose k unordered things from n things.
15. The expressions $\arcsin x$, $\sin^{-1} x$, $\arccos x$, $\cos^{-1} x$, $\arctan x$, $\tan^{-1} x$ refer to the principal values of these inverse trigonometric functions. This means that
$$-\frac{\pi}{2} \leq \sin^{-1} x \leq \frac{\pi}{2}, \quad 0 \leq \cos^{-1} x \leq \pi, \quad -\frac{\pi}{2} \leq \tan^{-1} x \leq \frac{\pi}{2}.$$
16. If a trigonometric problem does not specify the use of degrees, all trigonometric expressions are given in radians.
17. The floor function (or greatest integer function) is denoted by $\lfloor x \rfloor$, and it is defined as $\lfloor x \rfloor = n$ when n is an integer and $n \leq x < n + 1$. Similarly, the ceiling function (or least integer function) is denoted by $\lceil x \rceil$, and it is defined as $\lceil x \rceil = n$ when n is an integer and $n - 1 < x \leq n$.

18. The fractional part is denoted by $\{x\}$, and it is defined as $\{x\} = x - \lfloor x \rfloor$.
19. The Euler totient function is denoted by $\varphi(n)$, and it is defined as the number of positive integers less than or equal to n that are relatively prime to n . That is,

$$\varphi(n) = |\{k \in \mathbb{N} : 1 \leq k \leq n \text{ and } \gcd(k, n) = 1\}|.$$

20. Intervals are written as a pair of numbers. Round brackets indicate that the endpoint is excluded, while square brackets indicate that the endpoint is included. For example, the interval $(2, 3]$ denotes $\{x : 2 < x \leq 3\}$.
21. The greatest lower bound of a set is the largest number that is less than or equal to every number of the set. For example, the greatest lower bound of the intervals $(2, 3)$ and $[2, 3]$ are both 2. The least upper bound of a set is the smallest number that is greater than or equal to every number of the set. For example, the least upper bound of intervals $(2, 3)$ and $[2, 3]$ are both 3.
22. $\max(a_1, a_2, \dots, a_n)$ denotes the largest element in the set of real numbers $\{a_1, a_2, \dots, a_n\}$. Similarly, $\min(a_1, a_2, \dots, a_n)$ denotes the smallest element in the set of real numbers $\{a_1, a_2, \dots, a_n\}$.
23. The expression \sqrt{x} denotes the nonnegative square root of x .
24. Functions are assumed to be well-defined wherever they are used.
25. The absolute value of a real or complex number z is denoted by $|z|$.
26. The real part and the imaginary part of a complex number z are denoted by $\operatorname{Re} z$ and $\operatorname{Im} z$ respectively. If $z = a + bi$ where a and b are real, then $\operatorname{Re} z = a$ and $\operatorname{Im} z = b$.
27. When the complex logarithm is used, $\log(z)$ denotes the principal branch unless otherwise specified:

$$\log(z) = \ln |z| + i \operatorname{Arg}(z), \quad -\pi < \operatorname{Arg}(z) \leq \pi.$$

28. The empty set is denoted by \emptyset .
29. The empty product is equal to 1, and the empty sum is equal to 0.
30. An abelian group is a group such that the group operation is commutative.
31. Unless otherwise specified, a ring is assumed to have distinct additive and multiplicative identities; in particular, $0 \neq 1$.
32. If $q = p^k$ for a prime p and positive integer k , \mathbb{F}_q denotes the finite field with q elements.
33. The notation $f : A \rightarrow B$ denotes a function from A to B .
34. The determinant of a square matrix A is denoted by $\det(A)$.
35. For a matrix M , the entry in the i th row and j th column is denoted m_{ij} , and we equivalently denote the matrix by (m_{ij}) .
36. If π is a permutation of a finite set, its sign, denoted $\operatorname{sgn}(\pi)$, is defined by

$$\operatorname{sgn}(\pi) = \begin{cases} +1 & \text{if } \pi \text{ can be written as a product of an even number of transpositions,} \\ -1 & \text{if } \pi \text{ can be written as a product of an odd number of transpositions.} \end{cases}$$

A permutation with sign $+1$ is called even, and one with sign -1 is called odd.